STPM/S(E)954

PEPERIKSAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(*MALAYSIA HIGHER SCHOOL CERTIFICATE*)

MATHEMATICS T
Syllabus and Specimen Papers

This syllabus applies for the 2002 examination and thereafter until further notice. Teachers/candidates are advised to contact Majlis Peperiksaan Malaysia for the latest information about the syllabus.

MPM

MAJLIS PEPERIKSAAN MALAYSIA
(*MALAYSIAN EXAMINATIONS COUNCIL*)

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MAJLIS PEPERIKSAAN MALAYSIA
(*MALAYSIAN EXAMINATIONS COUNCIL*)
Bangunan MPM, Persiaran 1
Bandar Baru Selayang
68100 Batu Caves
Selangor Darul Ehsan

Telephone: 03-61369663
Facsimile: 03-61361488
E-mail: suagung@mpm.edu.my
Website: www.mpm.edu.my

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PERCETAKAN WARNI SDN BHD
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Tel: 03-61882666 (4 lines)
Fax: 03-61841402

FALSAFAH PENDIDIKAN KEBANGSAAN

“Pendidikan di Malaysia adalah suatu usaha berterusan ke arah memperkembangkan lagi potensi individu secara menyeluruh dan bersepadu untuk mewujudkan insan yang seimbang dan harmonis dari segi intelek, rohani, emosi, dan jasmani berdasarkan kepercayaan dan kepatuhan kepada Tuhan. Usaha ini adalah bagi melahirkan rakyat Malaysia yang berilmu pengetahuan, berketrampilan, berakhlak mulia, bertanggungjawab, dan berkeupayaan mencapai kesejahteraan diri serta memberi sumbangan terhadap keharmonian dan kemakmuran masyarakat dan negara”.

FOREWORD

Mathematics is a diverse and growing field of study. The study of mathematics can contribute towards clear, logical, quantitative and relational thinking, and also facilitates the implementation of programmes which require mathematical modeling, statistical analysis, and computer technology. Mathematics is finding ever wider areas of applications.

The mathematics syllabus for the Malaysia Higher School Certificate examination has been reviewed and rewritten so as to be more relevant to the current needs. The Mathematics T syllabus aims to develop the understanding of mathematical concepts and their applications, together with the skills in mathematical reasoning and problem solving, so as to enable students to proceed to programmes related to science and technology at institutions of higher learning.
The aims, objective, contents, form of examination, reference books, and specimen papers are presented in this booklet.

On behalf of Malaysian Examinations Council, I would like to thank the Malaysia Higher School Certificate Examination Mathematics Syllabus Committee chaired by Associate Professor Dr Harun bin Budin and all others who have contributed towards the development of this syllabus. It is hope that this syllabus will achieve its aims.

DATO' HAJI TERMUZI BIN HAJI ABDUL AZIZ
Chief Executive
Malaysian Examinations Council

CONTENTS

SYLLABUS

954 MATHEMATICS T

(May not be taken with 950 Mathematics S)

Aims

The Mathematics T syllabus aims to develop the understanding of mathematical concepts and their applications, together with the skills in mathematical reasoning and problem solving, so as to enable students to proceed to programmes related to science and technology at institutions of higher learning.

Objectives

The objectives of this syllabus are to develop the abilities of students to

(*a*) understand and use mathematical terminology, notation, principles, and methods;

(*b*) perform calculations accurately and carry out appropriate estimations and approximations;

(*c*) understand and use information in tabular, diagrammatic, and graphical forms;

(*d*) analyse and interpret data;

(*e*) formulate problems into mathematical terms and solve them;

(*f*) interpret mathematical results and make inferences;

(*g*) present mathematical arguments in a logical and systematic manner.

Content

1. Numbers and sets

1.1 Real numbers

1.2 Exponents and logarithms

1.3 Complex numbers

1.4 Sets

Explanatory notes

Candidates should be able to

(*a*) understand the real number system;

(*b*) carry out elementary operations on real numbers;

(*c*) use the properties of real numbers;

(*d*) use the notation for intervals of real numbers;

(*e*) use the notation $|x|$ and its properties;

(*f*) understand integral and rational exponents;

(*g*) understand the relationship between logarithms and exponents;

(*h*) carry out change of base for logarithms;

(*i*) use the laws of exponents and laws of logarithms;

(*j*) use the results: for $a > b$ and $c > 1$, $c^a > c^b$ and $\log_c a > \log_c b$; for $a > b$ and $0 < c < 1$, $c^a < c^b$ and $\log_c a < \log_c b$;

(*k*) solve equations and inequalities involving exponents and logarithms;

(*l*) understand the meaning of the real part, imaginary part, and conjugate of a complex number;

(*m*) find the modulus and argument of a complex number;

(*n*) represent complex numbers geometrically by means of an Argand diagram;

(*o*) use the condition for the equality of two complex numbers;

(*p*) carry out elementary operations on complex numbers expressed in cartesian form;

(*q*) understand the concept of a set and set notation;

(*r*) carry out operations on sets;

(*s*) use the laws of the algebra of sets.

2. Polynomials

2.1 Polynomials

2.2 Equations and inequalities

2.3 Partial fractions

Explanatory notes

Candidates should be able to

(*a*) understand the meaning of the degrees and coefficients of polynomials;

(*b*) carry out elementary operations on polynomials;

(*c*) use the condition for the equality of two polynomials;

(*d*) find the factors and zeroes of polynomials;

(*e*) prove and use the remainder and factor theorems;

(*f*) use the process of completing the square for a quadratic polynomial;

(*g*) derive the quadratic formula;

(*h*) solve linear, quadratic, and cubic equations and equations that can be transformed into quadratic or cubic equations;

(*i*) use the discriminant of a quadratic equation to determine the properties of its roots;

(*j*) prove and use the relationships between the roots and coefficients of a quadratic equation;

(*k*) solve inequalities involving polynomials of degrees not exceeding three, rational functions, and the modulus sign;

(*l*) solve a pair of simultaneous equations involving polynomials of degrees not exceeding three;

(*m*) express rational functions in partial fractions.

3. Sequences and series

3.1 Sequences

3.2 Series

3.3 Binomial expansions

Explanatory notes

Candidates should be able to

(*a*) use an explicit or a recursive formula for a sequence to find successive terms;

(*b*) determine whether a sequence is convergent or divergent and find the limit of a convergent sequence;

(*c*) use the Σ notation;

(*d*) use the formula for the general term of an arithmetic or a geometric progression;

(*e*) derive and use the formula for the sum of the first n terms of an arithmetic or a geometric series;

(*f*) use the formula for the sum to infinity of a convergent geometric series;

(*g*) solve problems involving arithmetic or geometric progressions or series;

(*h*) use the method of differences to obtain the sum of a finite or a convergent infinite series;

(*i*) expand $(a + b)^n$ where n is a positive integer;

(*j*) expand $(1 + x)^n$ where n is a rational number and $|x| < 1$;

(*k*) use the binomial expansion for approximation.

4. Matrices

4.1 Matrices

4.2 Inverse matrices

4.3 System of linear equations

Explanatory notes

Candidates should be able to

(*a*) understand the terms null matrix, identity matrix, diagonal matrix, and symmetric matrix;

(*b*) use the condition for the equality of two matrices;

(*c*) carry out matrix addition, matrix subtraction, scalar multiplication, and matrix multiplication for matrices with at most three rows and three columns;

(*d*) find the minors, cofactors, determinants, and adjoints of 2×2 and 3×3 matrices;

(*e*) find the inverses of 2×2 and 3×3 non-singular matrices;

(*f*) use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$;

(*g*) use inverse matrices for solving simultaneous linear equations;

(*h*) solve problems involving the use of a matrix equation.

5. Coordinate geometry

5.1 Cartesian coordinates in a plane

5.2 Straight lines

5.3 Curves

Explanatory notes

Candidates should be able to

(*a*) understand cartesian coordinates for the plane and the relationship between a graph and an associated algebraic equation;

(*b*) calculate the distance between two points and the gradient of the line segment joining two points;

(*c*) find the coordinates of the mid-point and the point that divides a line segment in a given ratio;

(*d*) find the equation of a straight line;

(*e*) use the relationships between gradients of parallel lines and between gradients of perpendicular lines;

(*f*) calculate the distance from a point to a line;

(*g*) determine the equation of a circle and identify its centre and radius;

(*h*) use the equations and graphs of ellipses, parabolas, and hyperbolas;

(*i*) use the parametric representation of a curve (excluding trigonometric expressions);

(*j*) find the coordinates of a point of intersection;

(*k*) solve problems concerning loci.

6. Functions

6.1 Functions and graphs

6.2 Composite functions

6.3 Inverse functions

6.4 Limit and continuity of a function

Explanatory notes

Candidates should be able to

(*a*) understand the concept of a function (and its notations) and the meaning of domain, codomain, range, and the equality of two functions;

(*b*) sketch the graphs of algebraic functions (including simple rational functions);

(*c*) use the six trigonometric functions for angles of any magnitude measured in degrees or radians;

(*d*) use the periodicity and symmetry of the sine, cosine, and tangent functions, and their graphs;

(*e*) use the functions e^x and $\ln x$, and their graphs;

(*f*) understand the terms one-one function, onto function, even function, odd function, periodic function, increasing function, and decreasing function;

(*g*) use the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{f}(x)|$;

(*h*) use the relationships between the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x + a)$, and $y = \mathrm{f}(ax)$;

(*i*) find composite and inverse functions and sketch their graphs;

(*j*) illustrate the relationship between the graphs of a one-one function and its inverse;

(*k*) sketch the graph of a piecewise-defined function;

(*l*) determine the existence and the value of the left-hand limit, right-hand limit, or limit of a function;

(*m*) determine the continuity of a function.

7. Differentiation

7.1 Derivative of a function

7.2 Rules for differentiation

7.3 Derivative of a function defined implicitly or parametrically

7.4 Applications of differentiation

Explanatory notes

Candidates should be able to

(*a*) understand the derivative of a function as the gradient of a tangent;

(*b*) obtain the derivative of a function from first principles;

(*c*) use the notations $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$;

(*d*) use the derivatives of x^n (for any rational number n), e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$;

(*e*) carry out differentiation of $k\mathrm{f}(x)$, $\mathrm{f}(x) \pm \mathrm{g}(x)$, $\mathrm{f}(x)\mathrm{g}(x)$, $\frac{\mathrm{f}(x)}{\mathrm{g}(x)}$, $(\mathrm{f} \circ \mathrm{g})(x)$;

(*f*) find the first derivative of an implicit function;

(*g*) find the first derivative of a function defined parametrically;

(*h*) find the gradients of and the tangents and normals to the graph of a function;

(*i*) find the intervals where a function is increasing or decreasing;

(*j*) understand the relationship between the sign of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and concavity ;

(*k*) determine stationary points, local extremum points, and points of inflexion (end-points of an interval where a function is defined are not regarded as stationary or local extremum points);

(*l*) determine absolute minimum and maximum values;

(*m*) sketch graphs (excluding oblique asymptotes);

(*n*) find an approximate value for a root of a non-linear equation by using the Newton-Raphson method;

(*o*) solve problems concerning rates of change, minimum values, and maximum values.

8. Integration

8.1 Integral of a function

8.2 Integration techniques

8.3 Definite integrals

8.4 Applications of integration

Explanatory notes

Candidates should be able to

(*a*) understand indefinite integration as the reverse process of differentiation;

(*b*) use the integrals of x^n (for any rational number n), e^x, $\sin x$, $\cos x$, $\sec^2 x$;

(*c*) carry out integration of $k\mathrm{f}(x)$ and $\mathrm{f}(x) \pm \mathrm{g}(x)$;

(*d*) integrate a function in the form $\{\mathrm{f}(x)\}^r\,\mathrm{f}'(x)$, where r is a rational number;

(*e*) integrate a rational function by means of decomposition into partial fractions;

(*f*) use substitutions to obtain integrals;

(*g*) use integration by parts;

(*h*) evaluate a definite integral, including the approximate value by using the trapezium rule;

(*i*) calculate plane areas and volumes of revolution about one of the coordinate axes.

9. Differential equations

9.1 Differential equations

9.2 First order differential equations with separable variables

9.3 First order homogeneous differential equations

Explanatory notes

Candidates should be able to

(*a*) understand the meaning of the order and degree of a differential equation;

(*b*) find the general solution of a first order differential equation with separable variables;

(*c*) find the general solution of a first order homogeneous differential equation;

(*d*) find the general solution of a differential equation which can be transformed into one the above types;

(*e*) sketch a family of solution curves;

(*f*) use the boundary condition to find a particular solution;

(*g*) solve problems that can be modelled by differential equations.

10. Trigonometry

10.1 Solution of a triangle

10.2 Trigonometric formulae

10.3 Trigonometric equations

Explanatory notes

Candidates should be able to

(*a*) use the sine and cosine rules;

(*b*) use the formulae $\Delta = \frac{1}{2}ab \sin C$ and $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$;

(*c*) solve problems in two or three dimensions;

(*d*) use the formulae $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$;

(*e*) derive and use the formulae for $\sin (A \pm B)$, $\cos (A \pm B)$, $\tan (A \pm B)$, $\sin A \pm \sin B$, $\cos A \pm \cos B$;

(*f*) express $a \sin \theta + b \cos \theta$ in the forms $r \sin (\theta \pm \alpha)$ and $r \cos (\theta \pm \alpha)$;

(*g*) find all solutions, within a specified interval, of a trigonometric equation or inequality.

11. Deductive geometry

11.1 Euclid's axioms

11.2 Polygons

11.3 Circles

Explanatory notes

Candidates should be able to

(*a*) understand Euclid's axioms and the results that follow, such as the properties of angles at a point, angles related to parallel lines, and angles of a triangle;

(*b*) prove and use the properties of plane figures, similar triangles, and congruent triangles;

(*c*) prove and use theorems about angles in a circle;

(*d*) prove and use theorems about chords and tangents;

(*e*) prove and use theorems about cyclic quadrilaterals.

12. Vectors

12.1 Vectors

12.2 Applications of vectors

Explanatory notes

Candidates should be able to

(*a*) understand the concept of a vector and its notations $\overrightarrow{AB}$, **a**, $\underset{\sim}{a}$, $\begin{pmatrix} x \\ y \end{pmatrix}$, and $x\mathbf{i} + y\mathbf{j}$;

(*b*) understand the terms unit vectors, parallel vectors, equivalent vectors, and position vectors;

(*c*) calculate the magnitude and direction of a vector;

(*d*) carry out addition and subtraction of vectors and multiplication of a vector by a scalar;

(*e*) use the properties of vectors, including $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$;

(*f*) use the scalar product to find the angle between two vectors and determine the perpendicularity of vectors;

(*g*) use vectors to prove geometrical results;

(*h*) solve problems concerning resultant forces, resultant velocities, and relative velocities.

13. Data description

13.1 Representation of data

13.2 Measures of location

13.3 Measures of dispersion

Explanatory notes

Candidates should be able to

(*a*) understand discrete, continuous, ungrouped, and grouped data;

(*b*) construct and interpret stemplots, boxplots, histograms, and cumulative frequency curves;

(*c*) derive and use the formula $\sum_{i=1}^{n}(x_i - \overline{x})^2 = \sum_{i=1}^{n} x_i^2 - n(\overline{x})^2$;

(*d*) estimate graphically and calculate measures of location and measures of dispersion;

(*e*) interpret the mode, median, mean, range, semi-interquartile range, and standard deviation;

(*f*) understand the symmetry and skewness in a data distribution.

14. Probability

14.1 Techniques of counting

14.2 Events and probabilities

14.3 Mutually exclusive events

14.4 Independent and conditional events

Explanatory notes

Candidates should be able to

(*a*) use counting rules for finite sets, including the inclusion-and-exclusion rule, for two or three sets;

(*b*) use the formulae for permutations and combinations;

(*c*) understand the concepts of sample spaces, events, and probabilities;

(*d*) understand the meaning of complementary and exhaustive events;

(*e*) calculate the probability of an event;

(*f*) understand the meaning of mutually exclusive events;

(*g*) use the formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$;

(h) understand the meaning of independent and conditional events;

(i) use the formula $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B \mid A)$.

15. Discrete probability distributions

15.1 Discrete random variables

15.2 Mathematical expectation

15.3 The binomial distribution

15.4 The Poisson distribution

Explanatory notes

Candidates should be able to

(a) understand the concept of a discrete random variable;

(b) construct a probability distribution table for a discrete random variable;

(c) understand the concept of the mathematical expectation;

(d) use the formulae $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$, $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$, and, for independent X and Y, $\mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$;

(e) derive and use the formula $\mathrm{E}(X - \mu)^2 = \mathrm{E}(X^2) - \mu^2$;

(f) calculate the mean and variance of a discrete random variable;

(g) understand the binomial and Poisson distributions;

(h) use the probability functions of the binomial and Poisson distributions;

(i) use the binomial and Poisson distributions as models for solving problems;

(j) use the Poisson distribution as an approximation to the binomial distribution, where appropriate.

16. Continuous probability distributions

16.1 Continuous random variables

16.2 Probability density function

16.3 Mathematical expectation

16.4 The normal distribution

Explanatory notes

Candidates should be able to

(a) understand the concept of a continuous random variable;

(b) understand the concept of a probability density function;

(c) use the relationship between the probability density function and the cumulative distribution function;

(d) understand the concept of the mathematical expectation;

(e) use the formulae $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$, $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$, and, for independent X and Y, $\mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$;

(f) derive and use the formula $\mathrm{E}(X - \mu)^2 = \mathrm{E}(X^2) - \mu^2$;

(*g*) calculate the mean and variance of a continuous random variable;

(*h*) solve problems which are modelled with appropriate probability density functions;

(*i*) understand the normal distribution;

(*j*) standardise a normal variable;

(*k*) use normal distribution tables;

(*l*) use the normal distribution as a model for solving problems;

(*m*) use the normal distribution as an approximation to the binomial distribution, where appropriate.

Form of Examination

The examination consists of two papers; the duration for each paper is 3 hours. Candidates are required to take both Paper 1 and Paper 2.

Paper 1 (same as Paper 1, Mathematics S) is based on topics 1 to 8 and Paper 2 is based on topics 9 to 16. Each paper contains 12 compulsory questions of variable mark allocations totalling 100 marks.

Reference Books

1. Bostock, L. & Chandler, S., C., *Core Maths for Advanced Level* (Third Edition), Nelson Thornes, limited, 2000.

2. Smedley, R. & Wiseman, G., *Introducing Pure Mathematics* (Second Edition), Oxford University Press, 2001.

3. Sullivan, M., *Algebra & Trigonometry* (Sixth Edition), Prentice Hall, 2002.

4. Stewart, J., *Calculus: Concepts and Contexts, Single Variable* (Second Edition), Brooks/Cole, 2001.

5. Crawshaw, J. & Chambers, J., *A Concise Course in Advanced Level Statistics* (Fourth Edition), Nelson Thornes Limited, 2001.

6. Johnson, R. A. & Bhattacharyya, G. K., *Statistics: Principles and Methods* (Fourth Edition), John Wiley & Sons, 2001.

7. Upton, G. & Cook, I., *Introducing Statistics* (Second Edition), Oxford University Press, 2001.

8. How, G. A. & Sim, J. T., *Siri Teks STPM: Matematik Tulen T*, Pearson Malaysia Sdn. Bhd., 2002.

9. Ong, B. S. & Abdul Aziz Jemain, *Matematik STPM Jilid 1: Tulen*, Penerbit Fajar Bakti Sdn. Bhd., 2001.

10. Tey, K. S., Tan, A. G., & Goh, C. B., *Matematik STPM: Matematik S & Matematik T – Kertas 1*, Penerbitan Pelangi Sdn. Bhd., 2001.

11. Khor, S. C., Heong, S. T., Tey, K. S., Goh, C. B., & Poh, A. H., *Matematik STPM: Matematik T – Kertas 2*, Penerbitan Pelangi Sdn. Bhd., 2002.

12. Soon C. L., Tong, S. F., & Lau, T. K., *Siri Teks STPM: Matematik: Statistik T*, Pearson Malaysia Sdn. Bhd., 2002.

13. Tan, C. E., Chew, C. B., Lye, M. S., & Abdul Aziz Jemain, *Matematik STPM Jilid 2: Tulen dan Statistik*, Penerbit Fajar Bakti Sdn. Bhd., 2001.

SPECIMEN PAPER

950/1, 954/1 STPM

MATHEMATICS S

PAPER 1

MATHEMATICS T

PAPER 1

(Three hours)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 By using the laws of set algebra, show that, for any sets A and B,

$$A \cap (A \cap B)' = A \cap B'.$$ [3 *marks*]

2 Solve the simultaneous equations

$$\log_4(xy) = \frac{1}{2},$$

$$(\log_2 x)(\log_2 y) = -2.$$ [6 *marks*]

3 Express $\dfrac{1}{(4r-3)(4r+1)}$ in partial fractions. Hence show that

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{1}{4}\left(1 - \frac{1}{4n+1}\right).$$ [6 *marks*]

4 Find the equation of the normal to the curve $x^2y + xy^2 = 12$ at the point (3, 1). [6 *marks*]

5 Evaluate the definite integral $\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$. [6 *marks*]

6 Show that the mid-points of the parallel chords of the parabola $y^2 = 4ax$ with gradient 2 lie on a straight line parallel to the x-axis. [7 *marks*]

7 The functions f and g are defined by

$$\mathrm{f}: x \mapsto 2x, \; x \in \mathbb{R};$$

$$\mathrm{g}: x \mapsto \cos x - |\cos x|, \; -\pi \le x \le \pi.$$

(i) Find the composite function $\mathrm{f} \circ \mathrm{g}$, and state its domain and range. [4 *marks*]

(ii) Show, by definition, that $\mathrm{f} \circ \mathrm{g}$ is an even function. [2 *marks*]

(iii) Sketch the graph of $\mathrm{f} \circ \mathrm{g}$. [2 *marks*]

8 Draw, on the same axes, the graphs of $y = \mathrm{e}^{-\frac{1}{2}x}$ and $y = 4 - x^2$. State the integer which is nearest to the positive root of the equation

$$x^2 + \mathrm{e}^{-\frac{1}{2}x} = 4.$$ [3 *marks*]

Find an approximation for this positive root by using the Newton-Raphson method until two successive iterations agree up to two decimal places; give your answer correct to two decimal places. [5 *marks*]

9 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 8 & 0 \\ 1 & 3 & 5 \end{pmatrix}, \; \mathbf{B} = \begin{pmatrix} -2 & 0 & 0 \\ -1 & -5 & 0 \\ -1 & -3 & -2 \end{pmatrix}.$$

(i) Determine whether **A** and **B** commute. [3 *marks*]

(ii) Show that there exist numbers m and n such that $\mathbf{A} = m\mathbf{B} + n\mathbf{I}$, where **I** is the 3×3 identity matrix, and find the values of m and n. [6 *marks*]

10 Given that $y = \dfrac{1}{\sqrt{1+2x}+\sqrt{1+x}}$ where $x > -\dfrac{1}{2}$, show that, provided $x \neq 0$,

$$y = \frac{1}{x}\left(\sqrt{1+2x}-\sqrt{1+x}\right).$$ [3 *marks*]

Using the second form for y, express y as a series of ascending powers of x as far as the term in x^2. [6 *marks*]

Hence, by putting $x = \dfrac{1}{100}$, show that

$$\frac{10}{\sqrt{102}+\sqrt{101}} \approx \frac{79\,407}{160\,000}.$$ [3 *marks*]

11 Show that the curve $y = \dfrac{\ln x}{x}$ has a stationary point at $\left(\mathrm{e}, \dfrac{1}{\mathrm{e}}\right)$, and determine whether this point is a local minimum point or a local maximum point. [6 *marks*]

Sketch the curve. [3 *marks*]

Show that the area of the region bounded by the curve $y = \dfrac{\ln x}{x}$, the x-axis, and the line $x = \dfrac{1}{\mathrm{e}}$ is equal to the area of the region bounded by the curve $y = \dfrac{\ln x}{x}$, the x-axis, and the line $x = \mathrm{e}$.

[5 *marks*]

12 Show that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Deduce that both roots are real if $b^2 - 4ac \geq 0$ and are complex if $b^2 - 4ac < 0$. [4 *marks*]

Determine all real values of k for which the quadratic equation

$$x^2 - (k-3)x + k^2 + 2k + 5 = 0$$

has real roots. [5 *marks*]

If α and β are the roots of this quadratic equation, show that $\alpha^2 + \beta^2 = -(k+5)^2 + 24$. Hence find the maximum value for $\alpha^2 + \beta^2$. [6 *marks*]

SPECIMEN PAPER

954/2 STPM

MATHEMATICS T

PAPER 2

(Three hours)

MAJLIS PEPERIKSAAN MALAYSIA
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SIJIL TINGGI PERSEKOLAHAN MALAYSIA
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Instructions to candidates:

Answer **all** *questions.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae, and graph paper are provided.

1 Find all values of θ, where $-\pi \le \theta \le \pi$, which satisfy the equation

$$\sin 4\theta - \sin 2\theta = \cos 3\theta.$$ [4 *marks*]

2 Forces $(4\mathbf{i} + 3\mathbf{j})$ N, $(3\mathbf{i} + 7\mathbf{j})$ N, and $(-5\mathbf{i} - 6\mathbf{j})$ N act at a point. Calculate the magnitude of the resultant force and the cosine of the angle between the resultant force and the unit vector $\mathbf{i}$. [5 *marks*]

3 The points A, B, and C are three points on the horizontal ground with B due north of A and the bearing of C from B being 060°. The angles of elevation of the top of a vertical tower situated at B from A and C are both β. The point P lies on AC such that $AP : PC = 1 : 2$. Show that the angle of elevation of the top of the tower from P is $\tan^{-1}\left(\sqrt{3}\tan\beta\right)$. [8 *marks*]

4 The position vectors of the points A, B, C, and D are **a, b, c,** and **d** respectively. If $ABCD$ is a rectangle, show that

$$|\mathbf{a}|^2 + |\mathbf{c}|^2 = |\mathbf{b}|^2 + |\mathbf{d}|^2.$$ [8 *marks*]

5 In a biochemical process, enzyme A changes continuously to enzyme B. Throughout the process, the total amount of A and B is constant. At any time, the rate that B is produced is directly proportional to the product of the amount of A and the amount of B at that time. At the beginning of the process, the amount of A and the amount of B are a and b respectively. If x denotes the amount of B that has been produced at time t after the process has begun, form a differential equation relating x and t to describe the process. [2 *marks*]

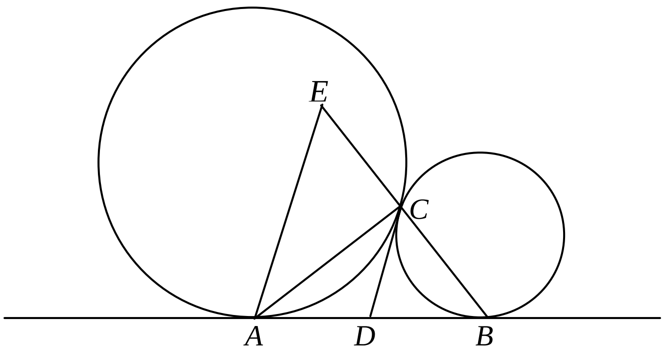
Show that the solution of the differential equation is

$$x = \frac{ab(1 - \mathrm{e}^{-(a+b)kt})}{b + a\mathrm{e}^{-(a+b)kt}},$$

where k is a positive constant. [8 *marks*]

Sketch the graph of x against t. [There is a point of inflection on the graph.] [2 *marks*]

6 Prove that the tangents from an external point to a circle are equal in length. [4 *marks*]



In the above figure, two circles touch externally at the point C. The points A and B are the points of contact between the circles and a common tangent, and the tangent at C meets AB at the point D. AE is parallel to DC and BCE is a straight line. Show that

(i) $AD = DB$, [3 *marks*]

(ii) $\angle ACB = 90°$, [4 *marks*]

(iii) $BC = CE$. [2 *marks*]

7 A random variable X has a Poisson distribution with $P(X = 0) = P(X = 1)$. Find $E(X^2)$. [5 *marks*]

8 One in a thousand foreign workers is known to have a certain disease. Result from a routine screening of a foreign worker may be positive or negative. A positive result suggests that the worker has the disease, but the test is not perfect. If a foreign worker has the disease, the probability of a negative result is 0.02. If a foreign worker does not have the disease, the probability of a positive result is 0.01. If the result of a test on a foreign worker is positive, find the probability that this worker has the disease. Comment on the answer you obtain. [6 *marks*]

9 Among 100 students in a school, 40 like lemons, 62 like mangosteens, 56 like nutmegs, 18 like lemons and nutmegs, 15 like lemons and mangosteens, 10 like all the three fruits, and 11 do not like any of the three fruits. Find the probability that

(i) a student chosen at random likes only lemons, [3 *marks*]

(ii) a student chosen at random likes mangosteens and nutmegs but does not like lemons. [4 *marks*]

10 The distance travelled by a newspaper vendor in a residential district for each weekday (Monday to Friday) has mean 13 km and standard deviation 0.8 km. For Saturdays and Sundays, the daily distance travelled has mean 11 km and standard deviation 0.7 km. The distances travelled on different days may be assumed to be independent.

If D is the average daily distance travelled by the newspaper vendor in a week, find $E(D)$ and $\text{Var}(D)$. Assuming that D has a normal distribution, find the probability that, in a randomly chosen week, the mean daily distance travelled by the newspaper vendor is less than 12 km. [7 *marks*]

11 A maufacturer produces a type of car battery with a lifetime of X years which is a random variable having the probability density function

$$f(x) = \begin{cases} ax^2 + bx - \frac{9}{4}, & 1 \le x \le 3, \\ 0, & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) It is found that 50 out of 100 batteries produced by the manufacturer have lifetimes of less than 2 years. Determine the values of a and b. [7 *marks*]

(ii) Find the probability that a battery produced by the manufacturer lasts more than 2 years and 4 months. [3 *marks*]

12 The data shown in the stemplot below are the marks in a statistics course obtained by a group of students at a local institution of higher learning.

3	1 4
4	0 2 3 7 9
5	1 2 2 6 8
6	1 3 3 4
7	0 2
8	2
9	1

Key: 9 | 1 means 91 marks

(i) Find the percentage of students who obtain less than 40 marks and the percentage of students who obtain at least 80 marks. [2 *marks*]

(ii) Find the mean and standard deviation of the students' marks. [5 *marks*]

(iii) Find the median and semi-interquartile range of the students' marks. [4 *marks*]

(iv) Construct a boxplot for the above data. [4 *marks*]